The speed of light need not be constant

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Recent observations of the fine structure of spectral lines in the early universe have been interpreted as a variation of the fine structure constant. From the assumed validity of Maxwell equations in general relativity and well known experimental facts, it is proved that e and $\hbar$ are absolute constants. On the other hand, the speed of light need not be constant.

There have recently been indications that the fine structure of spectral lines in the early universe differs from its usual properties [1]. This has been interpreted as a slow change in the fine structure constant over cosmological times, and there is a lively controversy [2] whether this is due to variations of e, $\hbar$, or c. In this note it is proved, from the validity of Maxwell equations in general relativity and well known experimental facts, that e and $\hbar$ are absolute constants. On the other hand, the speed of light need not be constant.

It is often stated that the speed of light is exactly 299 792 458 m/sec [3], owing to a 1983 decision of Comité International des Poids et Mesures on how to define the unit of length "meter" [4]. However, this administrative decision raises serious issues [5,6]. Such a way of defining length implies that there must be *synchronized* clocks at the extremities of the measured object in order to determine the corresponding time interval. Synchronization is best performed by light signals [7] so that we effectively have a circular definition. Einstein himself wrote, after the first equation of his historic paper, "we assume that this definition of synchronization is free from contradictions, and possible for any number of points." Contradictions naturally appear in non-inertial coordinate systems. For example, if we are serious in measuring distance by means of the elapsed time for light signals, then the circumference of the Earth at the equator is 123.9m longer if measured eastward than if measured westward [6].

Genuine gravitional fields cause even more difficulties. Again quoting Einstein [8]: "the constancy of the velocity of light can be maintained only insofar as one restricts oneself to spatio-temporal regions of constant gravitational potential." Serious practical difficulties indeed have to be overcome for setting the global positioning system (GPS) [9]. In summary, the legal value of c is not in general the actual speed of electromagnetic signals. The only meaning of this exact, invariant, official speed of light is to serve for writing Lorentz transformations, if there is any need of them.

Let us briefly examine some alternatives. Can $\hbar$ change? Planck's constant is more than just a conversion factor between Joules and Hertz. Quantum systems are not localized, they are pervasive. In particular, entangled systems may be spread over arbitrary distances. A value of $\hbar$ varying in spacetime would necessitate a complete revision of quantum theory. Can the electric charge vary? It is a historical accident that Coulomb's law of force between macroscopic charges was discovered before it was known that the electric charges of all particles are integral multiples of e (or $e/3$ if we include quarks). This indicates that we should define $e = 1$ as the unit of charge (this is a *natural* unit, not a conversion factor). A more formal proof of constancy will be given below.

We must therefore have a closer look at c. It cannot be a mundane conversion factor. Time is not a fourth dimension of space. Relativistic transformations never change the nature of timelike and spacelike intervals; important information is lost if we ignore the difference. Likewise, in the gravity field of the Earth, the vertical and horizontal directions are not equivalent. Airlines measure them in feet and miles, respectively. The use of different length units may be advantageous in dimensional analysis: it is easily seen that a trajectory with initial velocities (v_x, v_h) must be a parabola $h = av_h(x/v_x) + bg(x/v_x)^2$. (A complete dynamical calculation gives $a = 1$ and $b = -1/2$.)

In general relativity, all four coordinates may have different dimensions and the constant c does not appear at all in the fundamental equations (it may appear in particular solutions, once sources that are not generally covariant have been specified with arbitrary units). In the early universe, where background radiation cannot be ignored, Lorentz invariance does not hold. There is a preferred frame. It is then plausible that, in such an environment, the "vacuum" behaves as a dielectric medium where the speed of light is different from its ideal value [10].

In this paper, I shall not use the full machinery of general relativity, but only the invariance of the fundamental equations under arbitrary nonlinear coordinate transformations. Einstein's gravitational equations are not used, Newton's constant G does not appear, so that Planck units, whatever they mean, are not involved. The only assumption is the validity of Maxwell's equations

$$F_{\mu\nu,\rho} + F_{\nu\rho,\mu} + F_{\rho\mu,\nu} = 0, \tag{1}$$

and

$$\mathcal{F}^{\mu\nu}{}_{,\nu} = \mathcal{J}^{\mu}, \tag{2}$$

where the covariant antisymmetric tensor $F_{\mu\nu}$ corresponds to the field components usually called $\mathbf{E}$ and $\mathbf{B}$, and the contravariant antisymmetric tensor density $\mathcal{F}^{\mu\nu}$

corresponds to **D** and **H**. Commas denote partial derivatives (not covariant derivatives) and $\mathcal{J}^\mu$ is a vector density. The existence of a metric is not needed for the above form of Maxwell's equations. Lorentz invariance is irrelevant to them. The metric appears only in the relation between F and $\mathcal{F}$ which is, in vacuo,

$$\mathcal{F}^{\mu\nu} = \sqrt{-g}\, g^{\mu\rho}\, g^{\nu\sigma}\, F_{\rho\sigma}. \tag{3}$$

This form of Maxwell equations was first written by Einstein [11]. Later, they were again derived by Weyl [12] and Brillouin [13] who systematically developed the notion of densities and capacities. It has even been proposed to consider F and $\mathcal{F}$ as the fundamental fields, and the metric, defined by Eq. (3), as a derived quantity [14–16].

An apparent difficulty, which will actually turn into a powerful tool, is the fact that spacetime coordinates may have different dimensions (time, length, angles, and so on). Then likewise the components of F have different dimensions, and also those of $\mathcal{F}$. The notion of *absolute dimension* of a tensor was introduced by Schouten [17] and Post [18]. As a simple example, consider the electric charge

$$Q = \int \mathcal{J}^\mu\, dS_\mu, \tag{4}$$

where dS_μ is an element of a three-dimensional hypersurface, such as $dxdydz$ or $drd\theta d\phi$. Therefore dS_μ transforms as a covariant vector capacity, so that Q is invariant under arbitrary coordinate dimensions, and we say that the absolute dimension of $\mathcal{J}$ is [Q]. It is also the absolute dimension of $\mathcal{F}$, owing to Eq. (2). It follows that the electronic charge e is invariant, as we already found by a more intuitive reasoning.

As we have seen, dimensional analysis is an important tool for studying nature and deriving useful results. Dimensional analysis also applies to absolute dimensions: they must be the same on both sides of any equation. Consider now Eq. (1). It states that there are no magnetic charges and guarantees the local existence of a covariant vector field A_μ such that

$$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}. \tag{5}$$

For any given closed path, the value of the magnetic flux,

$$\Phi = \oint A_\mu\, dx^\mu, \tag{6}$$

is a scalar (i.e., it is invariant under any coordinate transformations). It follows that the absolute dimension of F is that of magnetic flux, namely [action/charge]. Now, it is experimentally known that in some superconductors [19], flux appears in integral multiples of $h/2e$. This is the natural unit of flux, just as e is the natural unit of charge. It follows that $\hbar$ is an absolute constant, in agreement with the intuitive argument presented above.

There is no similar argument for the constancy of c (or for mass ratios of elementary particles, and other important "constants"). It is plausible that in the early universe, when matter and radiation were much more concentrated than today, atomic energy levels were different from those of truly isolated atoms, in a way analogous to the chemical shift of nuclear energy levels in the Mössbauer effect [20]. However this phenomenon is not yet understood, and more work, theoretical and experimental, is needed. We may then hope that more precise observations on the fine structure of spectral lines will give information on the properties of the cosmic environment in early universe.

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[1] J. K. Webb *et al.*, Phys. Rev. Lett. **82**, 884 (1999); **87**, 091301 (2001).
[2] M. J. Duff, e-print hep-th/0208092; J. W. Moffat, e-print hep-th/0208109; J. D. Bekenstein, e-print gr-qc/0208081; J. D. Barrow, e-print gr-qc/0209080; and references therein.
[3] P. J. Mohr and B. N. Taylor, Physics Today **55** (8) BG6 (2002).
[4] P. Giacomo, Am. J. Phys. **52**, 607 (1984).
[5] B. W. Petley, Nature **303**, 373 (1983); J. Phys. E **17**, 823 (1984).
[6] A. Peres, Nature **312**, 10 (1984).
[7] A. Einstein, Ann. Physik **17**, 891 (1905).
[8] A. Einstein, Ann. Physik **38**, 1059 (1912).
[9] N. Ashby, Physics Today **55** (5) 41 (2002).
[10] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1960).
[11] A. Einstein, Sitzungsber. Königl. Preuss. Akad. Wiss. (1916) p. 184.
[12] H. Weyl, Sitzungsber. Königl. Preuss. Akad. Wiss. (1918) p. 465; *Raum Zeit Materie* (Springer, Berlin, 1921).
[13] L. Brillouin, *Les Tenseurs en Mécanique et en Elasticité* (Masson, Paris, 1949) chap. 3.
[14] A. Peres, Ann. Physics (NY) **19**, 279 (1962).
[15] Yu. N. Obukhov and F. W. Hehl, Physics Letters B **458**, 466 (1999); F. W. Hehl, Yu. N. Obukhov, and G. F. Rubilar, Ann. Physik **11** (special issue S1) 71 (2000).
[16] F. W. Hehl and Yu. N. Obukhov, in *Gyros, Clocks, Interferometers. . . : Testing Relativistic Gravity in Space*, C. Lämmerzahl *et al.*, eds.. Lecture Notes in Physics Vol. 562 (Springer, Berlin, 2001) pp. 479–504; *Foundations of Classical Electrodynamics* (Birkhäuser, Boston, 2003); e-print physics/0005084.
[17] J. A. Schouten, *Tensor Analysis for Physicists* (Oxford Univ. Press, Oxford, UK, 1951).

[18] E. J. Post, *Formal Structure of Electromagnetics* (North-Holland, Amsterdam, 1962) chap. 2.
[19] P.-G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966) chap. 5.
[20] H. Frauenfelder and E. M. Henley, *Subatomic Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1974) pp. 498–503.
[21] A. Peres, e-print quant-ph/0209114.